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MATEMATIKA
MATHEMATICS

Izpitna pola 1 / *Question Paper 1*

Vzorec / 120 minut
Sample / 120 minutes

Dovoljeno gradivo in pripomočki:
Kandidat prinese nalivno pero ali kemični svinčnik, svinčnik, radirko, računalo in geometrijsko orodje
(šestilo in dva trikotnika, lahko tudi ravnilo).
Kandidat dobi dva konceptna lista in obrazec za vrednotenje.

Items and materials allowed:
Candidates should have a fountain pen or a ballpoint pen, a pencil, an eraser, a calculator and a geometry set – a pair of
compasses, two triangles and a ruler (optional).
Candidates receive two draft sheets and a marking sheet.

IZPIT ZA OSEBE Z MEDNARODNO ZAŠČITO
EXAM FOR PERSONS WITH INTERNATIONAL PROTECTION

Ta pola ima 20 strani, od tega 1 rezervno in 3 prazne.
This Question Paper contains 20 pages, 1 of which is spare and 3 of which are blank.

NAVODILA KANDIDATU

Pazljivo preberite ta navodila.

Ne odpirajte izpitne pole in ne začenjajte reševati nalog, dokler vam nadzorni učitelj tega ne dovoli.

Prilepite kodo oziroma vpišite svojo šifro (v okvirček desno zgoraj na prvi strani in na obrazec za vrednotenje). Svojo šifro vpišite tudi na konceptna lista.

Izpitna pola vsebuje 12 kratkih nalog. Število točk, ki jih lahko dosežete, je 80. Za posamezno nalogo je število točk navedeno v izpitni poli. Pri reševanju si lahko pomagate s standardno zbirko zahtevnejših formul na strani 3.

Rešitve, ki jih pišite z nalivnim peresom ali s kemičnim svinčnikom, vpisujte v izpitno polo **v za to predvideni prostor**. Rišete lahko tudi s svinčnikom. Če se zmotite, napisano prečrtajte in rešitev zapišite na novo. Nečitljivi zapisi in nejasni popravki bodo ocenjeni z 0 točkami. Stran 17 je rezervna; uporabite jo le, če vam zmanjka prostora. Jasno označite, katere naloge ste reševali na tej strani. Osnutki rešitev, ki jih lahko naredite na konceptna lista, se pri ocenjevanju ne upoštevajo.

Pri reševanju nalog mora biti jasno in korektno predstavljena pot do rezultata z vsemi vmesnimi računi in sklepi. Če ste nalogo reševali na več načinov, jasno označite, katero rešitev naj ocenjevalec oceni.

Zaupajte vase in v svoje zmožnosti. Želimo vam veliko uspeha.

INSTRUCTIONS TO CANDIDATES

Read these instructions carefully.

Do not open the Question Paper and do not start doing the test questions until the invigilator allows it.

Stick the label with your barcode, or write your number, in the space provided in the upper right-hand corner on the front page and on the marking sheets. Write your number on the draft sheets.

There are 12 short tasks. The total number of points is 80. The number of points awarded for each task is indicated in the Question Paper. You can refer to the Formula Sheet on p 4 for more complex formulas needed to do the test questions.

*Complete with a fountain pen or a ballpoint pen **in the spaces provided**. You can use the pencil for drawing. If you make a mistake, cross it out, and write the new answer next to it. Illegible answers and unclear corrections will be awarded 0 points. Page 17 is a spare page; use it only if you run out of space, and indicate clearly which test questions you did on this page. Drafts for solutions – which you can write on the draft sheets – will not be evaluated.*

In solving the tasks, the path to the result with all interim calculations and conclusions must be clearly and correctly presented. If you attempted to do a test question in more than one way, clearly indicate which solution should be assessed.

Believe in yourself and your abilities. We wish you every success.

Formule

Evklidov in višinski izrek v pravokotnem trikotniku: $a^2 = ca_1, \ b^2 = cb_1, \ v_c^2 = a_1 b_1$

Polmera trikotniku očrtanega in včrtanega kroga: $R = \dfrac{abc}{4S}, \ r = \dfrac{S}{s}, \ s = \dfrac{a+b+c}{2}$

Adicijski izrek:

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

Razdalja točke $T_0(x_0, y_0)$ od premice $ax + by - c = 0$: $\ d(T_0, p) = \left| \dfrac{ax_0 + by_0 - c}{\sqrt{a^2 + b^2}} \right|$

Ploščina trikotnika z oglišči $A(x_1, y_1), \ B(x_2, y_2), \ C(x_3, y_3)$:

$S = \dfrac{1}{2} \left| (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1) \right|$

Elipsa: $e^2 = a^2 - b^2, \varepsilon = \dfrac{e}{a}, \ $ če je $a > b$

Hiperbola: $e^2 = a^2 + b^2$

Parabola: $y^2 = 2px$, gorišče $G\left(\dfrac{p}{2}, 0 \right)$

Kompozitum funkcij: $(g \circ f)(x) = g(f(x))$

Integral: $\displaystyle \int \dfrac{dx}{x^2 + a^2} = \dfrac{1}{a} \arctan \dfrac{x}{a} + C$

The Formula Sheet

Euclid's theorem and the right triangle altitude theorem: $a^2 = ca_1, \; b^2 = cb_1, \; v_c^2 = a_1 b_1$

The radii of a circumscribed and an inscribed circle in a triangle: $R = \dfrac{abc}{4S}, \; r = \dfrac{S}{s}, \; s = \dfrac{a+b+c}{2}$

Addition theorem:

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Distance of the point $T_0\left(x_0,\, y_0\right)$ *from the line* $ax + by - c = 0$: $\; d\left(T_0,\, p\right) = \left| \dfrac{ax_0 + by_0 - c}{\sqrt{a^2 + b^2}} \right|$

The area of a triangle with vertices $A(x_1, y_1), \; B(x_2, y_2), \; C(x_3, y_3)$:

$$S = \frac{1}{2}\left| (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1) \right|$$

Ellipse: $e^2 = a^2 - b^2, \; \varepsilon = \dfrac{e}{a}, \;$ *if* $a > b$

Hyperbola: $e^2 = a^2 + b^2$

Parabola: $y^2 = 2px$, *focus* $G\left(\dfrac{p}{2}, 0\right)$

Composite function: $(g \circ f)(x) = g\left(f(x)\right)$

Integral: $\displaystyle\int \dfrac{\mathrm{d}x}{x^2 + a^2} = \dfrac{1}{a}\arctan\dfrac{x}{a} + C$

1. Poenostavite izraz $A = \left((-a)^4\right)^3 \cdot (-a)^{-3} : a^9, \ \ a \neq 0,$ do oblike, iz katere je razvidno, da je vrednost izraza neodvisna od a.

   Simplify the expression $A = \left((-a)^4\right)^3 \cdot (-a)^{-3} : a^9, \ \ a \neq 0,$ to a form from which it can be seen that the value of the expression is independent from a.

   (5 točk/points)

2. Nalogo rešujte **brez** uporabe računala.

   Rešite enačbi:

   *Do the test questions **without** using a calculator.*

   Solve the equations:

   2.1. $\log_x \dfrac{5}{3} = -1$

   <div align="right">*(2)*</div>

   2.2. $3^x + 3^{x+2} = \dfrac{10}{9}$

   <div align="right">*(4)*
(6 točk/points)</div>

3. V aritmetičnem zaporedju a_1, a_2, 2, a_4, 8 ... izračunajte a_1, a_2, a_4, a_{671} in vsoto prvih 671 členov.

   In the arithmetic sequence a_1, a_2, 2, a_4, 8 ... calculate a_1, a_2, a_4, a_{671} and the sum of its first 671 terms.

   (8 točk/points)

4. Pokažite, da je število -4 ena od ničel polinoma $p(x) = x^3 + 6x^2 + 10x + 8$, $x \in \mathbb{R}$. Poiščite preostali dve ničli polinoma p. Zapišite presečišče N grafa polinoma p z ordinatno osjo. Točka T leži na grafu polinoma p in ima absciso -1. Zapišite točko T.

Let $p(x) = x^3 + 6x^2 + 10x + 8$, for $x \in \mathbb{R}$. Show that -4 is one of the zeros of the given polynomial p. Find the other two zeros of p. Write down the intersection point N of the graph of p with the y-axis. Point T lies on the graph of p and its x-coordinate is -1. Find point T.

(7 točk/points)

5. Nalogo rešujte **brez** uporabe računala.

Dano je kompleksno število $z = \sqrt{5} - 2i$. Izračunajte:

*Do the test questions **without** using a calculator.*

Given the complex number $z = \sqrt{5} - 2i$, calculate:

5.1. $z \cdot \overline{z} =$

(2)

5.2. $|z| =$

(1)

5.3. $z^2 + i^{19} =$

(3)

5.4. $z^{-1} =$

(2)
(8 točk/points)

6. Iz skupine 7 fantov in 5 deklet naključno izberemo 4 osebe. Izračunajte verjetnost dogodka A, da bodo izbrani trije fantje in eno dekle.
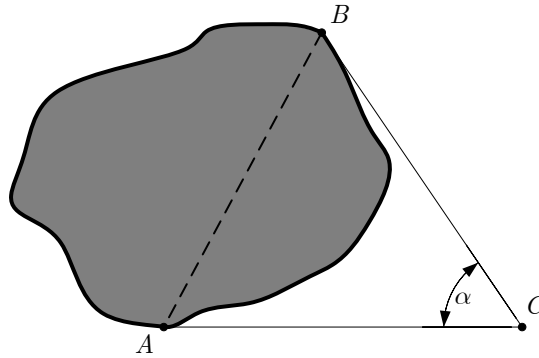
   From a group of 7 boys and 5 girls we randomly choose 4. Calculate the probability of event A that 3 boys and 1 girl are chosen.

   (7 točk/points)

7. Dobili ste nalogo, da izračunate širino jezera med točkama A in B. Izmerili ste $|AC| = 255$ m, $|BC| = 232$ m in $\alpha = 56°$. Kolikšna je razdalja med točkama A in B? Rezultat zaokrožite na meter natančno.

You are given the task of finding the width of the lake between points A and B. Your measurements are: $|AC| = 255$ m, $|BC| = 232$ m and $\alpha = 56°$. What is the distance between points A and B? Give your answer to the nearest metre.

(5 točk/points)

8. V pravokotnem koordinatnem sistemu so dane točke $A(2, 1)$, $B(-2, 3)$ in $C(3, -2)$. Zapišite vektorja $\overrightarrow{AB}$ in $\overrightarrow{AC}$ s komponentami, izračunajte njun skalarni produkt in kot, ki ga oklepata.

Consider the points $A(2, 1)$, $B(-2, 3)$ and $C(3, -2)$. Find vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Calculate their dot product. Hence, find the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(8 točk/points)

9. Zapišite enačbo krožnice, ki poteka skozi izhodišče koordinatnega sistema, njeno središče pa je v presečišču premic $2x - 3y - 9 = 0$ in $y + 1 = 0$.
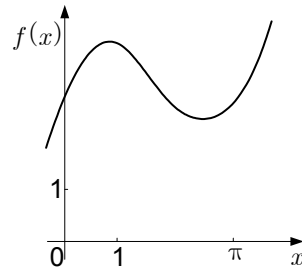
   Write an equation of the circle that passes through the origin and has its centre in the intersection point of the lines $2x - 3y - 9 = 0$ and $y + 1 = 0$.

   (6 točk/points)

10. Naj bo $f(x) = 2x + 3\cos x$, $x \in \mathbb{R}$. Na sliki spodaj je del grafa funkcije f. Izračunaj ploščino lika med grafom funkcije f, x osjo in premicama $x = 0$ in $x = \pi$.

   Let $f(x) = 2x + 3\cos x$, for $x \in \mathbb{R}$. Part of the graph of f is given on the diagram below. Calculate the area of the region enclosed by the graph of f, the x axis and the lines $x = 0$ and $x = \pi$.

(6 točk/points)

11. Vsota dolžin katet pravokotnega trikotnika je 56, dolžina njegove hipotenuze je 40. Izračunajte dolžini katet.

The length of the hypotenuse in a right-angled triangle is 40. The sum of the lengths of the other two sides is 56. Find the lengths of the other two sides.

(6 točk/points)

12. Racionalna funkcija f ima predpis $f(x) = \dfrac{x^2 + 3}{x + 1}$, $x \neq -1$. Zapišite točki $E_1(x_1,\ y_1)$ in $E_2(x_2,\ y_2)$, ki sta lokalna ekstrema funkcije f. V kateri točki ima funkcija lokalni minimum in v kateri lokalni maksimum? Odgovor utemeljite.

The rational function f is defined as $f(x) = \dfrac{x^2 + 3}{x + 1}$, $x \neq -1$. Function f has its local extrema at points $E_1(x_1,\ y_1)$ and $E_2(x_2,\ y_2)$. Find $E_1(x_1,\ y_1)$ and $E_2(x_2,\ y_2)$. In which point is the function's local minimum and in which is its local maximum? Justify your answer.

(8 točk/points)

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