Šifra kandidata:
Candidate number:

Državni izpitni center

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MATEMATIKA
MATHEMATICS

Izpitna pola 1 / *Question Paper 1*

A) Kratke naloge / *Short tasks*
B) Krajše strukturirane naloge / *Short structured tasks*

Vzorec / 90 minut (30 + 60)
Sample / 90 minutes (30 + 60)

Dovoljeno gradivo in pripomočki:
Kandidat prinese nalivno pero ali kemični svinčnik, svinčnik, radirko in geometrijsko orodje (šestilo in ravnilo, lahko tudi trikotnik). Priloga s formulami in konceptna lista so na perforiranih listih, ki jih kandidat pazljivo iztrga. Kandidat dobi obrazec za vrednotenje.

Items and materials allowed:
Candidates should have a fountain pen or a ballpoint pen, a pencil, an eraser and a geometry set – a pair of compasses, a ruler and a triangle (optional). The Formula Sheet and both draft sheets are enclosed on perforated sheets so candidates can carefully tear them out. Candidates receive a marking sheet.

IZPIT ZA OSEBE Z MEDNARODNO ZAŠČITO
EXAM FOR PERSONS WITH INTERNATIONAL PROTECTION

Navodila kandidatu so na naslednji strani.
Instructions to candidates are on p 2.

Ta pola ima 20 strani, od tega 2 rezervni.
This Question Paper contains 20 pages, 2 of which are spare.

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NAVODILA KANDIDATU

Pazljivo preberite ta navodila.

Ne odpirajte izpitne pole in ne začenjajte reševati nalog, dokler vam nadzorni učitelj tega ne dovoli.

Pri reševanju te izpitne pole uporaba računala ni dovoljena.

Prilepite kodo oziroma vpišite svojo šifro v okvirček desno zgoraj na prvi strani in na obrazec za vrednotenje.

Izpitna pola je sestavljena iz dveh delov, dela A in dela B. Časa za reševanje je 90 minut. Priporočamo vam, da za reševanje dela A porabite 30 minut, za reševanje dela B pa 60 minut.

Izpitna pola vsebuje 8 kratkih nalog v delu A in 6 krajših strukturiranih nalog v delu B. Število točk, ki jih lahko dosežete, je 60, od tega 20 v delu A in 40 v delu B. Za posamezno nalogo je število točk navedeno v izpitni poli. Pri reševanju si lahko pomagate s standardno zbirko zahtevnejših formul na strani 3.

Rešitve, ki jih pišite z nalivnim peresom ali s kemičnim svinčnikom, vpisujte v izpitno polo **v za to predvideni prostor**. Rišete lahko tudi s svinčnikom. Če se zmotite, napisano prečrtajte in rešitev zapišite na novo. Nečitljivi zapisi in nejasni popravki bodo ocenjeni z 0 točkami. Strani 13 in 20 sta rezervni; uporabite ju le, če vam zmanjka prostora. Jasno označite, katere naloge ste reševali na teh straneh. Osnutki rešitev, ki jih lahko naredite na konceptna lista, se pri vrednotenju ne upoštevajo.

Pri reševanju nalog mora biti jasno in korektno predstavljena pot do rezultata z vsemi vmesnimi računi in sklepi. Če ste nalogo reševali na več načinov, jasno označite, katero rešitev naj ocenjevalec oceni.

Zaupajte vase in v svoje zmožnosti. Želimo vam veliko uspeha.

INSTRUCTIONS TO CANDIDATES

Read these instructions carefully.

Do not open the Question Paper and do not start doing the test questions until the invigilator allows it.

You must not use a calculator for this Question Paper.

Stick the label with your barcode, or write your number, in the space provided in the upper right-hand corner on the front page and on the marking sheet.

The Question Paper consists of two parts, Part A and Part B. You have 90 minutes to complete the Question Paper. It is recommended that you spend 30 minutes for answering test questions from Part A and 60 minutes for answering test questions from Part B.

There are 8 short tasks in Part A and 6 short structured tasks in Part B. The total number of points is 60 – 20 in Part A and 40 in Part B. The number of points awarded for each task is indicated in the Question Paper. You can refer to the Formula Sheet on p 4 for more complex formulas needed to do the test questions.

Complete with a fountain pen or a ballpoint pen ***in the spaces provided****. You can use the pencil for drawing. If you make a mistake, cross it out, and write the new answer next to it. Illegible answers and unclear corrections will be awarded 0 points. Pages 13 and 20 are spare pages; use them only if you run out of space, and indicate clearly which test questions you did on these pages. Drafts for solutions – which you can write on the draft sheets – will not be evaluated.*

In solving the tasks, the path to the result with all interim calculations and conclusions must be clearly and correctly presented. If you attempted to do a test question in more than one way, clearly indicate which solution should be assessed.

Believe in yourself and your abilities. We wish you every success.

Formule

(**Vsota in razlika kubov**) Za poljubna $a,\ b \in \mathbb{R}$ velja $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$.

(**Evklidov in višinski izrek**) Pravokotni trikotnik ima kateti a in b ter hipotenuzo c. Višina na hipotenuzo je v_c, pravokotna projekcija katete a na hipotenuzo je a_1, pravokotna projekcija katete b na hipotenuzo pa b_1. Tedaj velja $a^2 = ca_1,\ b^2 = cb_1$, $v_c^2 = a_1 b_1$.

(**Polmera trikotniku včrtanega in očrtanega kroga**) Trikotnik ima stranice a, b in c, polovica obsega je $s = \frac{a+b+c}{2}$, ploščina je S, polmer danemu trikotniku včrtanega kroga je r in polmer danemu trikotniku očrtanega kroga je R. Tedaj je $r = \frac{S}{s}$ in $R = \frac{abc}{4S}$.

(**Heronova formula**) Trikotnik ima stranice a, b in c, polovica obsega je $s = \frac{a+b+c}{2}$. Tedaj je njegova ploščina $S = \sqrt{s(s-a)(s-b)(s-c)}$.

(**Ploščina trikotnika**) Naj bodo $A(x_1,\ y_1)$, $B(x_2,\ y_2)$ in $C(x_3,\ y_3)$ točke v ravnini. Ploščina trikotnika z oglišči A, B in C je $S = \frac{1}{2}\left|(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)\right|$.

(**Krogla**) Površina in prostornina krogle s polmerom r sta $P = 4\pi r^2,\ V = \frac{4\pi r^3}{3}$.

(**Adicijski izreki**) Za poljubna $x,\ y \in \mathbb{R}$ velja

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$, $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$.

Za poljubna $x,\ y \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + \pi \cdot k;\ k \in \mathbb{Z}\right\}$, za katera je $x + y \neq \frac{\pi}{2} + \pi \cdot k$ za poljuben $k \in \mathbb{Z}$ in $\tan x \tan y \neq -1$, velja $\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$.

(**Kotne funkcije polovičnih kotov**)

Za poljuben $x \in \mathbb{R}$ velja $\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$, $\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$.

Za poljuben $x \in \mathbb{R} \setminus \{\pi + \pi \cdot 2k;\ k \in \mathbb{Z}\}$ velja $\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}$.

(**Elipsa**) Elipsa v ravnini ima polosi a in b $(a > b)$, njena linearna ekscentričnost je e, njena numerična ekscentričnost je ε. Tedaj velja $e^2 = a^2 - b^2$, $\varepsilon = \frac{e}{a}$.

(**Hiperbola**) Hiperbola v ravnini ima realno polos a in imaginarno polos b, njena linearna ekscentričnost je e, njena numerična ekscentričnost je ε. Tedaj velja $e^2 = a^2 + b^2$, $\varepsilon = \frac{e}{a}$.

(**Parabola**) Parabola v ravnini z enačbo $y^2 = 2px$ ima gorišče v $G\left(\frac{p}{2},\ 0\right)$, enačba premice vodnice dane parabole pa je $x = -\frac{p}{2}$.

(**Aritmetično zaporedje**) Vsota prvih n členov aritmetičnega zaporedja (a_n) je $S_n = \frac{n}{2}(a_1 + a_n)$.

(**Geometrijsko zaporedje**) Vsota prvih n členov geometrijskega zaporedja (a_n) s kvocientom $q \in \mathbb{R}$ je $S_n = \frac{a_1(q^n - 1)}{q - 1}$, če je $q \neq 1$, in $S_n = na_1$, če je $q = 1$.

(**Limiti**) $\lim\limits_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$ in $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$.

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The Formula Sheet

(Sum and difference of cubes) *For any* $a,\ b \in \mathbb{R}$ *the following identities hold true* $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$.

(Euclidean and altitude theorem) *A right-angled triangle has a hypotenuse* c. *The catheti are* a *and* b. *The altitude on the hypotenuse is* v_c *and the projections of the catheti* a *and* b *on the hypotenuse are* a_1 *and* b_1 *respectively. Then* $a^2 = ca_1,\ b^2 = cb_1$, $v_c^2 = a_1 b_1$.

(Radii of the inscribed and circumscribed circle of a triangle) *A triangle has sides* a, b *and* c. *The semiperimeter is denoted by* $s = \frac{a+b+c}{2}$. *The area of the triangle is* S. *The radius of the inscribed circle is* r *and the radius of the circumscribed circle is* R. *Then* $r = \frac{S}{s}$ *and* $R = \frac{abc}{4S}$.

(Heron's formula) *A triangle has sides* a, b *and* c. *The semiperimeter is denoted by* $s = \frac{a+b+c}{2}$. *The area of the triangle is* S. *Then* $S = \sqrt{s(s-a)(s-b)(s-c)}$.

(Area of a triangle) *Let* $A(x_1,\ y_1)$, $B(x_2,\ y_2)$ *and* $C(x_3,\ y_3)$ *be points on a plane. The area* S *of a triangle with vertices* A, B *and* C *is* $S = \frac{1}{2}\left|(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)\right|$.

(Sphere) *The surface area* P *and the volume* V *of a sphere with radius* r *are* $P = 4\pi r^2,\ V = \frac{4\pi r^3}{3}$.

(Trigonometric addition formulas) *For any* $x,\ y \in \mathbb{R}$ *the following identities hold true* $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$, $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$.

For any $x,\ y \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + \pi \cdot k;\ k \in \mathbb{Z}\right\}$, *such that* $x + y \neq \frac{\pi}{2} + \pi \cdot k$, $k \in \mathbb{Z}$ *and* $\tan x \tan y \neq -1$, *the following identity holds true* $\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$.

(Trigonometric half angle formulas)

For any $x \in \mathbb{R}$ *the following identities hold true* $\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$, $\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$.

For any $x \in \mathbb{R} \setminus \{\pi + \pi \cdot 2k;\ k \in \mathbb{Z}\}$ *the following identity holds true* $\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}$.

(Ellipse) *Let* a *and* b $(a > b)$ *be semiaxes of an ellipse on a plane. Linear eccentricity of an ellipse is denoted by* e *and numerical eccentricity of an ellipse is denoted by* ε. *Then* $e^2 = a^2 - b^2$, $\varepsilon = \frac{e}{a}$.

(Hyperbola) *Let* a *be a real semiaxis and let* b *be an imaginary semiaxis of a hyperbola on a plane. Linear eccentricity of a hyperbola is denoted by* e *and numerical eccentricity of a hyperbola is denoted by* ε. *Then* $e^2 = a^2 + b^2$, $\varepsilon = \frac{e}{a}$.

(Parabola) *A parabola on a plane with an equation* $y^2 = 2px$ *has a focus in point* $G\left(\frac{p}{2},\ 0\right)$. *The equation of the directrix of a parabola is* $x = -\frac{p}{2}$.

(Arithmetic sequence) *The sum of the first* n *terms of an arithmetic sequence* (a_n) *is* $S_n = \frac{n}{2}(a_1 + a_n)$.

(Geometric sequence) *The sum of the first* n *terms of a geometric sequence* (a_n) *with a common ratio* $q \in \mathbb{R}$ *is* $S_n = \frac{a_1(q^n - 1)}{q - 1}$ *if* $q \neq 1$, *and* $S_n = na_1$ *if* $q = 1$.

(Limits) $\lim\limits_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$ *and* $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$.

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A) KRATKE NALOGE / *SHORT TASKS*

1. Označite pravilno nadaljevanje stavkov.
   Premici z enačbama $4x+y-3=0$ in $y=-4x+2021$

   – sta vzporedni.
   – nista vzporedni.

   Sistem enačb $4x+y=3$ in $4x+y=2021$

   – nima rešitve.
   – ima natanko eno rešitev.
   – ima neskončno mnogo rešitev.

   Indicate which ending of the sentence is correct.
   The lines given by equations $4x+y-3=0$ *and* $y=-4x+2021$

   – *are parallel.*
   – *are not parallel.*

   The system of equations $4x+y=3$ *and* $4x+y=2021$

   – *does not have a solution.*
   – *has exactly one solution.*
   – *has infinitely many solutions.*

   (2 točki/points)

2. Izračunajte $0{,}2\overline{6}\cdot 0{,}75$ in rezultat zapišite v obliki okrajšanega ulomka.

   Calculate $0{,}2\overline{6}\cdot 0{,}75$. *Give your answer in a form of fraction reduced to its simplest form.*

   (2 točki/points)

3. Delno korenite in izračunajte $\sqrt{20}-\sqrt{80}+3\sqrt{125}$.

Simplify the square roots and calculate $\sqrt{20}-\sqrt{80}+3\sqrt{125}$.

(2 točki/points)

4. Poenostavite algebrski izraz $\left(\frac{x^2-1}{x^2}\right)^{-1}\cdot\left(x^0-x^{-2}\right)$.

Simplify the algebraic expression $\left(\frac{x^2-1}{x^2}\right)^{-1}\cdot\left(x^0-x^{-2}\right)$.

(3 točke/points)

5. V množici kompleksnih števil rešite enačbo $z^2 - 2z + 5 = 0$.

Solve the equation $z^2 - 2z + 5 = 0$ in the set of complex numbers.

(2 točki/points)

6. Daljica ima krajišči $A(3,\ 3)$ in $B(0,\ 7)$. Izračunajte njeno dolžino in zapišite koordinati razpolovišča daljice.

Points $A(3,\ 3)$ and $B(0,\ 7)$ are the end points of a line segment AB. Find the lenght of the line segment AB and the coordinates of the midpoint of the line segment AB.

(3 točke/points)

7. Izračunajte $\frac{\sin 60° + \tan(-45°)}{\cos \frac{21\pi}{4}}$.

Rezultat zapišite v obliki $\frac{2\sqrt{m} - \sqrt{n}}{2}$, kjer sta $m, n \in N$.

Calculate the value of the expression $\frac{\sin 60° + \tan(-45°)}{\cos \frac{21\pi}{4}}$.

Give your answer in the form $\frac{2\sqrt{m} - \sqrt{n}}{2}$, *where* $m, n \in N$.

(3 točke/points)

8. Izračunajte, pri katerem x je tangenta na graf funkcije $f(x) = e^x$ vzporedna premici $y = 3x + 2021$.

Find the x *coordinate of the point where the tangent line to the graph of the function* $f(x) = e^x$ *is parallel to the line* $y = 3x + 2021$.

(3 točke/points)

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B) KRAJŠE STRUKTURIRANE NALOGE / *SHORT STRUCTURED TASKS*

1. Poenostavite izraz $A=\left((-a)^4\right)^3\cdot(-a)^{-3}:a^9,\quad a\neq 0,$ do oblike, iz katere je razvidno, da je vrednost izraza neodvisna od a.

   Simplify the expression $A=\left((-a)^4\right)^3\cdot(-a)^{-3}:a^9,\quad a\neq 0,$ *to a form from which it can be seen that the value of the expression is independent from* a.

   (5 točk/points)

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2. Rešite enačbi:

Solve the equations:

2.1. $\log_x \frac{5}{3} = -1$

(2)

2.2. $3^x + 3^{x+2} = \frac{10}{9}$

(4)

(6 točk/points)

A 2 1 0 4 0 1 1 1 1 6

3. Dano je kompleksno število $z=\sqrt{5}-2\mathrm{i}$. Izračunajte:

Given the complex number $z=\sqrt{5}-2\mathrm{i}$, *calculate:*

3.1. $z\cdot\overline{z}=$

(2)

3.2. $|z|=$

(1)

3.3. $z^2+\mathrm{i}^{19}=$

(3)

3.4. $z^{-1}=$

(2)

(8 točk/points)

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4. Iz skupine 7 fantov in 5 deklet naključno izberemo 4 osebe. Izračunajte verjetnost dogodka A, da bodo izbrani 3 fantje in 1 dekle.

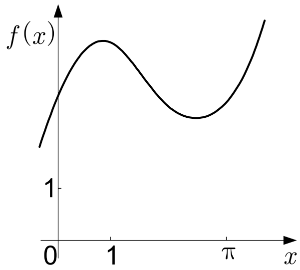
   From a group of 7 boys and 5 girls we randomly choose 4. Calculate the probability of event A that 3 boys and 1 girl are chosen.

(7 točk/points)

A 2 1 0 4 0 1 1 1 1 8

5. Naj bo $f(x) = 2x + 3\cos x$, $x \in \mathbb{R}$. Na sliki spodaj je del grafa funkcije f. Izračunaj ploščino lika med grafom funkcije f, x osjo in premicama $x = 0$ in $x = \pi$.

Let $f(x) = 2x + 3\cos x$, for $x \in \mathbb{R}$. Part of the graph of f is given on the diagram below. Calculate the area of the region enclosed by the graph of f, the x axis and the lines $x = 0$ and $x = \pi$.



(6 točk/points)

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6. Racionalna funkcija f ima predpis $f(x)=\dfrac{x^2+3}{x+1}$, $x\neq -1$. Zapišite točki $E_1(x_1,\ y_1)$ in $E_2(x_2,\ y_2)$, ki sta lokalna ekstrema funkcije f. V kateri točki ima funkcija lokalni minimum in v kateri lokalni maksimum? Odgovor utemeljite.

The rational function f is defined as $f(x)=\dfrac{x^2+3}{x+1}$, $x\neq -1$. Function f has its local extrema at points $E_1(x_1,\ y_1)$ and $E_2(x_2,\ y_2)$. Find $E_1(x_1,\ y_1)$ and $E_2(x_2,\ y_2)$. In which point is the function's local minimum and in which is its local maximum? Justify your answer.

(8 točk/points)

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