Annex 1

Ljubljana 2021

MATHEMATICS

Examination Guide for Persons with International Protection

INDEX

1 INTRODUCTION

The Examination Guide for Persons with International Protection – MATHEMATICS (hereinafter referred to as the *Guide*) defines the Mathematics exam as required by the *Decree on the methods and conditions for ensuring the rights of persons with international protection*. The aim of the *Guide* is to help candidates prepare for the assessment of Mathematics required for enrolment in tertiary education.

Candidates taking the Mathematics exam have to prove that they are capable of achieving the exam objectives as defined by this *Guide*.

The *Guide* is based on the Mathematics syllabus* and the *Subject Examination Guide for the General Matura Examination – Mathematics* for 2021. The contents and the objectives of the exam correspond to Mathematics at Basic Level in upper secondary education.

* Učni načrt. Matematika [Elektronski vir]: gimnazija: splošna, klasična in strokovna gimnazija: obvezni predmet in matura (560 ur)/predmetna komisija Amalija Žakelj … [et al.]. - Ljubljana: Ministrstvo za šolstvo in šport: Zavod RS za šolstvo, 2008. http://portal.mss.edus.si/msswww/programi2012/programi/gimnazija/ucni_nacrti.htm

2 AIM OF THE EXAM

In the Mathematics exam, candidates are expected to demonstrate that they can:

- read mathematical texts and correctly interpret them;
- clearly present mathematical contents in text, table, graph or diagram format;
- compute with numbers, evaluate and calculate the result with precision, as well as judge the result's validity;
- use the adequate method for calculating;
- apply information and communication technology (ICT) in solving mathematical problems;
- use the geometry set for drawing;
- interpret, reformulate and properly use mathematical statements, expressed either in words or in symbols;
- recognise and apply relationships between geometric objects (in the plane and in space);
- come to logical conclusions from given mathematical data;
- recognise patterns and structures in different situations;
- analyse a problem and choose the correct manner of solving it;
- notice and make use of the connections of different branches (areas) of mathematics;
- apply a combination of several mathematical skills and techniques in solving problems;
- present mathematical work in a logical and clear manner, using adequate symbols and terminology;
- apply mathematical knowledge in real-life situations;
- use mathematics as a means of communication with the emphasis on precise formulations.

3.1 Exam format

Question Paper	Time allowed	Weight	Assessment	Items allowed and required tools	Appendix
1	90 minutes	50 %	External	A fountain pen or a ballpoint pen, a pencil, an eraser and a geometry set[+]	The Formula Sheet
2	90 minutes	50 %	External	A fountain pen or a ballpoint pen, a pencil, an eraser, a calculator[*] and a geometry set[+]	The Formula Sheet
Total	**180 minutes**	**100 %**			

Question Paper 1 is followed by a 30-minute break.

3.2 Test questions types and marking

3.2.1 Test questions types

Question Paper	Type of task	No. of tasks	Marking
1	A Short tasks	8	Up to 3 points for each correct answer total 20 points
	B Short structured tasks	6	5 to 8 points for each correct answer total 40 points Total 60 points
2	A Short tasks	8	Up to 3 points for each correct answer total 20 points
	B Short structured tasks	6	5 to 8 points for each correct answer total 40 points Total 60 points
Total			**120 points**

[+] A pair of compasses, a ruler and a triangle (optional).

[*] A calculator is an electronic device used for performing basic arithmetic operations and should not support:
– communication with the environment – the 'outside world',
– storing data from the environment, or the 'outside world',
– storing previously uploaded data,
– computing with symbols,
– programming new functions,
– drawing graphs of functions.

3.2.2 Taxonomy Levels

Taxonomy Levels	Question Papers 1 and 2
I. knowledge	Min 30 %
II. comprehension and application	40–60 %
III. comprehension and application, solution of new problems	Max 30 %
Total	**100 %**

3.2.3 Criteria for assessment

Tasks are assessed in accordance with the Mark Scheme. Points are awarded for individual steps in the procedure that can be from different levels of taxonomy. In solving the tasks, the path to the result with all interim calculations and conclusions must be clearly and correctly presented. In mathematical constructions, candidates are required to use the geometry set.

3.3 Criteria for conversion of percentage points into a descriptive mark

The exam is marked by the Subject Committee for Mathematics in absolute and in percentage points. The points are then converted by the Subject Committee for Mathematics into a descriptive mark: either ‘Pass’ or ‘Fail’. Candidates pass the exam if they meet the criteria for ‘Pass’ in Mathematics in the General Matura Examination in the preceding calendar year.

4.1 Basic concepts of logic

Content	Objectives
	Candidates
Statements and relations between them Compound statements Order of operations Tautology Equivalent statements	– write a statement, – determine the truth-value of a statement, – write a compound statement using symbols, – determine the truth-value of a compound statement for all values of equivalent statements, – determine equivalence of two statements.

4.2 Sets

Content	Objectives
	Candidates
Basic concepts: element, set, set membership, subset, empty set, universal set Symbolic representations Venn diagram Intersection, union, difference, complement of sets Power sets Cartesian product of sets Cardinality of a set	– are familiar with basic concepts and mark relations between elements and sets using symbols, – use different methods for representations of sets, – compute with sets, – find the power set of a finite set, – draw the graph of a Cartesian product of two sets, – use formulas for the power of a union of two or three sets as well as the power of the Cartesian product of finite sets.

4.3 Number sets

Content	Objectives
4.3.1 Positive integers and integers	
	Candidates
Mathematical operations and their properties Prime numbers and composite numbers Decimal notation Criteria of divisibility by 2, 3, 4, 5, 6, 8, 9 and 10	– are familiar with the significance of positive integers and the reasons for the introduction of integers as well as examples of their use, – use mathematical operations in the set of positive integers and the set of integers, and can provide examples illustrating their properties,

Content	Objectives
Divisibility relation The greatest common divisor and the least common multiple Euclidean division theorem Decimal positional numeral system	– present positive integers and integers on a number line, – use decimal notation of whole numbers, – justify and use the basic criteria of divisibility, – are familiar with the properties of the divisibility relation and are able to apply them, – determine the greatest common divisor and the least common multiple of two or more integers, – use the Euclidean division theorem of integers;
4.3.2 Rational numbers	
Mathematical operations and their properties Decimal notation of rational numbers Proportions and percentage Percentage calculus	– are familiar with the reasons for the introduction of rational numbers and are able to justify them, – present rational numbers on a number line, – calculate with rational numbers, – use and explain a decimal notation of a rational number and distinguish between decimal and non-decimal fractions, – calculate with decimal numbers, – use proportions and percent as well as percentage calculus in tasks related to everyday life and are adept at using a calculator;
4.3.3 Real numbers	
Irrational numbers Real numbers on the number line Intervals Finite decimal approximations Absolute value of a real number and its properties Absolute value equations Absolute and relative error	– are familiar with the reasons for the introduction of real numbers and are able to justify them, – provide some examples of irrational numbers, – construct square roots as examples of irrational numbers using the Pythagorean theorem, – interpret the number line as a real axis, – round decimal numbers, – link geometric and analytical interpretations of the absolute value of real numbers, – simplify expressions with absolute value and solve simple equations, – compare the significance of absolute and relative errors and estimate absolute and relative errors of a sum, a difference, a product and a quotient of two data;

Content	Objectives

4.3.4 Complex numbers

Geometric representation of complex numbers in the plane Mathematical operations and their properties Solving equations with real coefficients	– are familiar with the reasons for the introduction of complex numbers and are able to justify them, – present a complex number in the complex plane, – use analytical and graphical methods to add and subtract complex numbers, – multiply complex numbers, – derive a rule for commuting powers of i, – find links between the analytical and geometric meaning of a complex conjugate, – find links between the analytical and the geometric significance of the absolute value of a complex number, – derive and apply the rule for division of complex numbers, – calculate the reciprocal of a complex number, – find complex solutions of equations.

4.4 Algebraic expressions, equations and inequalities

Content	Objectives
	Candidates
Mathematical operations with expressions Powers of expressions Factoring expressions Calculating with fractions Equations and inequalities Linear equation Decomposable form equation Linear inequality	– compare and distinguish between the notation for, and the significance of, an expression and an equation as well as a variable and an unknown, – add and multiply algebraic expressions, – apply and justify the rules on how to square and cube a binomial, – using Pascal's triangle, formulate the rules for higher powers of a binomial and use them, – recognise and use an adequate method of factoring a given expression: factoring out a common factor, the difference of squares, the sum and difference of cubes, Vieta's formulas, factoring quadrinomials, – calculate using algebraic fractions (all four mathematical operations and expressions with brackets), – apply rules for transforming equations to equivalent equations and effectively solve them,

Content	Objectives
	– recognise and solve linear equations, – recognise equations which can be solved by factoring and solve them, – effectively express unknowns from different equations from physics and chemistry, – apply rules for transforming inequalities to equivalent inequalities and effectively solve them, – recognise and solve linear inequalities.

4.5 Powers and roots

Content	Objectives
	Candidates
Powers with natural exponents Powers with integer exponents *n*th roots Powers with rational exponents	– justify and apply the rules for computing with power functions with natural exponents, – justify and apply the rules for computing with power functions with integer exponents and compare them to the rules for computing with power functions with natural exponents, – explain the significance of notations a^{-1} and a^{-n}, – apply the rules for computing with square roots, – solve quadratic equation of a form $x^2 = a,\ a > 0,\ a \in \mathbb{R}$ by factoring and determining square roots, – compare and explain solving of simple equations of a form $x^n = a,\ a \in \mathbb{R},\ n \in \mathbb{N}$ in a set of real numbers by determining square roots and factoring, – explain and use the relation $\sqrt{x^2} = \lvert x \rvert$, – compute exact cube roots of real numbers by heart (i.e., without aid) and using a calculator, – distinguish between various conditions for determination of existence of an *n*th root of a real number (with respect to the degree of root and the radicand), – are adept at using a calculator for computing *n*th roots, – transform the notation of an *n*th root into the notation of a power with rational exponents, – make links and compare solving tasks with *n*th roots to solving with powers with rational exponents.

4.6 Geometry in the plane and in space

Content	Objectives
	Candidates
Points, lines and circles in the plane Distance, a line segment, segment spanning a line, a bisector, a ray, an angle Types of angle and relationships between angles Triangle, polygons Famous points of a triangle Isometries and congruence Translation, reflection, rotation, orientation of a triangle Orthogonal projection Inscribed and central angle Angle in a semicircle Homothety, similarity Theorems in a right-angled triangle Parallelogram, rhombus, trapezium Mathematical constructions The sine and cosine rules Parallel and perpendicular lines and planes in three dimensional space Orthogonal projection of a line onto a plane	– understand concepts of elementary Euclidean geometry, – develop perception of geometry and, through practice, learn the basic standards of the mathematical theory, – are familiar with the definitions and apply the properties of geometric shapes, – apply relationships between interior and exterior angles of a triangle as well as relationships between sides and angles in a triangle, – apply the relationship between inscribed and central angles subtending the same arc, – distinguish between congruent and similar triangles, – apply theorems in a right-angled triangle, – construct shapes by using a pair of compasses, a ruler and a triangle (optional), – understand and apply relationship between sides and angles in an arbitrary triangle applying the sine and cosine rules, – explore geometric problems using ICT, – develop perception of relationships between points, lines and planes in space.

4.7 Geometric shapes and solids

Content	Objectives
	Candidates
Areas of geometric shapes, Heron's formula Radii of an inscribed and of a circumscribed circle in a triangle Geometric solids: prism, cylinder, pyramid, cone and sphere Surface area and volume of an upright prism, cylinder, pyramid, cone and sphere Geometric mathematical problems	– develop and improve perception of geometry, – express quantities from formulas, – estimate and critically evaluate the calculated values and pay attention to the units of measurement, – apply acquired knowledge of plane geometry and solve problems related to the radius of an inscribed and of a circumscribed circle in a triangle, – describe a geometry solid, – apply acquired knowledge of trigonometric functions and geometry on models of geometry solids,

Content	Objectives
	– solve geometric problems related to the surface area and the volume of a solid and estimate and critically evaluate the calculated results and the units of measurement, – recognise a geometric problem, present it, determine which concepts, variables and relationships between them can be applied to solve it, solve the problem, present solutions and considers its implications, – independently choose and apply appropriate strategies to solve geometric problems and link contents from plane geometry and space geometry in solving geometric problems, – solve geometric problems using trigonometry.

4.8 Vectors in the plane and in space

Content	Objectives
	Candidates
Definition of vectors Addition and scalar multiplication (forces) – graphic interpretation Collinearity, coplanarity – graphic interpretation Expressing vectors in a basis (writing a vector as a sum of components), Cartesian coordinate system – graphic interpretation Linear combination of vectors Basis in the plane and in space Cartesian coordinate system in the plane and in space; position vector of a point Notation of a vector in coordinates Mathematical operations with vectors expressed in coordinates Projection of a vector onto another vector Dot product, an angle between two vectors and the magnitude of a vector The relationship between the dot product and the cosine rule	– draw vectors, graphically add vectors and write a vector as a sum of two vectors, – learn how to operate with vectors graphically and algebraically, – evaluate collinearity and coplanarity of vectors, – operate with vectors expressed in coordinates, – calculate the angle between two vectors, the magnitude of a vector and orthogonal projection of a vector, – discuss perpendicular and parallel vectors, – understand perpendicularity in space.

4.9 Cartesian coordinate system in the plane

Content	Objectives
	Candidates
Sets of points in the plane Distance between two points in a coordinate plane	– use a Cartesian coordinate system in the plane,

Content	Objectives
Area of a triangle	– read and draw a set of points in the coordinate plane in given conditions, – apply the relationship between ordered pairs of numbers and points in the plane, – calculate the distance between two points with given coordinates, calculate the area of a triangle with given coordinates of the verices and use the two formulas for solving mathematical problems.

4.10 Functions

Content	Objectives
	Candidates
Definition of a function Definition of a real function and properties of real functions of real variables (injection, surjection, bijection, increasing and decreasing functions, even and odd functions…) Function composition Inverse function Transformations in the plane Limit of a function Special examples of limits Continuity of functions	– understand and use the expression of a function, – understand and use the expressions: domain and range of a function, injective, surjective and bijective functions, – draw and analyse the graph of a function by using translations, reflections, stretches or shrinks, – use translations, reflections, stretches or shrinks in solving problem-based tasks, – establish the existence of an inverse function on simple examples, offer its definition and draw the graph of an inverse function to the given function, – draw the graph of a piecewise-defined function, – explain the concept of the limit of a function at a given point with carefully chosen examples where functions are presented analytically or by their graphs or by the table of some of its values, – calculate the limit of a function at a given point and explain the significance of the calculated limit value, – explain the significance of the limit of a function at infinity, – distinguish between the limit of a function at infinity and the infinite limit of a function, – use limits in calculating asymptotes of functions, – recognise continuity of a function presented by its graph,

Content	Objectives
	– find intervals where a given function is continuous;
4.10.1 Linear function	
Definition and properties of a linear function, the graph of a linear function Equations of a line in the plane Angle between two lines Linear equation Linear inequality System of linear equations Modelling of simple examples from everyday life using a linear function	– define linear functions and draw their graphs, – are familiar with and apply the significance of coefficients in a linear function, – interpret and use the graph of a linear function in real-life situations, – calculate the angle between two lines, – are familiar with the significance of different forms of an equation of a line, – recognise linear relationships between variables and write a linear equation from a given text, – solve linear equations, – express a problem as a system of linear equations and solve it, – solve simple problems from everyday life and adequately interpret them, – model simple problems from everyday life using a linear function;
4.10.2 Power function	
Definition and properties of a power function with natural exponents Definition and properties of a power function with negative integer exponents Modelling of examples from everyday life using a power function	– recognise a power-dependence relation and distinguish it from other types of dependency relations (inverse proportionality…), – draw and analyse the graph of a power function using transformations, – formulate and model real-life phenomena using a power function and critically choose them;
4.10.3 Radical function	
Definition, properties and the graph of a radical function	– treat a radical function as the inverse function of a power function;
4.10.4 Quadratic function	
Definition, properties and the graph of a quadratic function Definition of a quadratic function and its equivalent forms	– find a quadratic function from different data and draw its graph, – interpret and use the graph of quadratic function in real-life situations,

Content	Objectives
Vieta's formulas Quadratic equation Intersection of a parabola and a line Intersection of two parabolas Quadratic inequality	– solve quadratic equations and quadratic inequalities, – translate a problem into an equation or an inequality and solve it, – read mathematical texts, analyse and present them;
4.10.5 Exponential function	
Definition, properties and the graph of an exponential function Exponential equations Exponential growth Modelling real-life phenomena using an exponential function	– recognise exponential dependence and distinguish it from other types of dependency relations, – are familiar with and apply the properties of an exponential function, – draw the graph of an exponential function, – use translations, reflections, stretches and shrinks of the graph of an exponential function, – compare power and exponential growth, – recognise and solve exponential equations, – find and model examples from everyday life using exponential functions;
4.10.6 Logarithmic function	
Definition, properties and the graph of a logarithmic function Logarithm and the rules of logarithmic computation The common logarithm and the natural logarithm Logarithmic equations	– are familiar with and apply the properties of a logarithmic function, – draw the graph of a logarithmic function, – apply the relationship between exponential and logarithmic functions, – use translations, reflections, stretches and shrinks of the graph of a logarithmic function, – apply the rules of logarithmic computation, – recognise the number e and the natural logarithm, – recognise and solve logarithmic equations, – compare exponential and logarithmic growth;
4.10.7 Polynomial function	
Definition, properties and the graph of a polynomial function Mathematical operations with polynomials Euclidean division of polynomials theorem Zeros of a polynomial function	– recognise linear and quadratic functions as special examples of polynomial functions, – compute with polynomials, – apply the Euclidean division of polynomials theorem,

Content	Objectives
The fundamental theorem of algebra and its corollaries Synthetic division of polynomials Analysis of the graph of a polynomial function Polynomial equations Polynomial inequalities	– apply the polynomial remainder theorem, – use synthetic division for finding zeros of a polynomial function, – apply the properties of polynomials in problem-based tasks, – draw and interpret the graph of a polynomial function, – solve polynomial equations and inequalities;

4.10.8 Rational function

Definition, properties and the graph of rational functions Zeros, poles and asymptotes Rational equations	– are familiar with and apply the properties of rational functions, – draw and interpret the graph of a rational function, – solve rational equations;

4.10.9 Trigonometric function

Definitions and properties of trigonometric functions in a right-angled triangle Definitions of trigonometric functions using a unit circle Properties and graphs of trigonometric functions Transformations of graphs of trigonometric functions Addition formulas or angle sum and difference identities Problem-based tasks Finding values of circular functions Trigonometric equations	– define and apply trigonometric functions in a right-angled triangle, – derive values of trigonometric functions for $0°$, $30°$, $45°$, $60°$, $90°$ angles, – derive and apply relationships between trigonometric functions of the same angle, – use a calculator, – use values of trigonometric functions for random angles, – are familiar with and apply the properties of trigonometric functions, – are familiar with and explain concepts in different modes of representation (table of values, a graph, using a unit circle, analytically), – apply transformations of graphs of trigonometric functions, – draw and interpret graphs of trigonometric functions, – apply addition formulas or apply angle sum and difference identities, – apply trigonometric functions of double angles, – use trigonometric functions of double angles in trigonometric equations and problem-based tasks, – calculate values of circular functions,

Content	Objectives
	– solve trigonometric equations, – interpret and analyse analytical solutions with regard to a given problem, – apply trigonometric functions in real-life situations where an angle has to be calculated, – solve simple, complex, authentic and original problems.

4.11 Conic sections

Content	Objectives
	Candidates
Algebraic notation of degree 2 curves Circle with a centre at the origin or with the centre at an arbitrary point $S(p,q)$ Ellipse with a centre at the origin or with the centre at an arbitrary point $S(p,q)$ Hyperbola with a centre at the origin Parabola with a vertex at the origin	– find examples of cone sections in nature, – compare and use analytic and geometric definitions of a cone section, – interpret a circle as a special example of an ellipse, – analyse equations and graphically present circles and ellipses centred at the origin and not centred at the origin, – analyse equations and graphically present hyperbolas and parabolas in vertex form, – analyse different forms of the equations of parabolas, – analytically and graphically determine intersections of a cone section and a line and determine intersections of cone sections centred at the origin, – explain the implications of results in analytical treatment of intersections.

4.12 Sequences and series

Content	Objectives
	Candidates
Definition of a sequence Properties of sequences (monotonous sequences, bounded sequences, convergent sequences…) Arithmetic sequence Geometric sequence The sum of first n terms of an arithmetic sequence and the sum of n terms geometric sequence	– provide an example, induce, generalise and continue a sequence, – find and write down the relationship between terms of a sequence, – continue the sequence which is given by a recursion, – determine and analyse the properties of sequences in different modes of representation (numerical, graphic and analytical representations…),

Content	Objectives
Limit of a sequence Series Convergence of a geometric series Percentage calculus Annuity Amortisation schedule	– find examples of sequences given or represented in different manners, – apply the properties of sequences in solving mathematical problems, – predict and calculate the limit of a sequence, – distinguish between a series and a sequence, – distinguish between a convergent and a divergent series, – compute the sum of n terms of a sequence, – compute the sum of a geometric series, – distinguish between simple and compound interest, – distinguish between conform and relative interest rate, – apply the equivalence of balance, – find real-life examples of interest, predict expectations and make decisions based on simulative calculations, – calculate annuity and make amortisation plan.

4.13 Differential calculus

Content	Objectives
	Candidates
Differential quotient, derivative, geometric interpretation of a derivative Differentiation rules, derivatives of elementary functions Application of the derivative Extreme values, increasing and decreasing differentiable functions on intervals Optimization problems	– describe concepts of differential calculus using graphic, numerical or analytical representations, – calculate the value of a differential quotient, – calculate the limit of a differential quotient, – explain geometric significance of a derivative, – derive elementary functions and composite functions, – determine points from the graph of a function where the function is not differentiable, – link the properties of a function and its derivative (predict properties, sketch a graph…), – write down the equations of a tangent and a normal in a given point of a curve, – calculate the angle between two curves,

Content	Objectives
	– analyse a function with the derivative (explain extremes, determine intervals of increase and decrease) and draw a graph, – solve simple optimization problems.

4.14 Integral calculus

Content	Objectives
	Candidates
Indefinite integral and primitive function Properties of indefinite integral Definite integral Properties of definite integral Relationship between definite and indefinite integrals Use of definite integral (areas)	– explain the relationship between the derivative of a function and the indefinite integral of a function, – are familiar with the table of basic integrals and its link to the table of derivatives, – apply the properties of an indefinite integral, – are familiar with geometric significance of a definite integral, – apply the properties of a definite integral in solving mathematical problems, – apply the relationship between a definite and an indefinite integral in solving mathematical problems, – solve simple mathematical and real problems using integrals.

4.15 Combinatorics

Content	Objectives
	Candidates
Fundamental theorem of combinatorics, tree diagrams The rule of sum Permutations Permutations with repetition Variations Variations with repetition Combinations Binomial theorem Pascal's triangle	– calculate $n!$, – distinguish between individual combinatorial concepts, – calculate the value of a binomial symbol, – expand a binomial raised to a power.

4.16 Probability

Content	Objectives
	Candidates
Fundamentals of probability: trial, event, the sample space	– formulate events and calculate with them,
Calculating the probability of events	– find all events for a trial,
Subjective probability, empirical probability, mathematical probability, probability of an event	– distinguish between subjective, empirical and mathematical probability,
Calculating the probability of opposite events, sums of events	– understand and link empirical and mathematical probability,
Normal distribution	– are familiar with and can apply the definition of mathematical probability,
	– from given probabilities of individual events calculate the probability of other events,
	– use the sample space.

4.17 Statistics

Content	Objectives
	Candidates
Basic statistical concepts	– distinguish between the studied properties (a variable), one element of the population, a value of a variable, a sample, a population,
Types of data	– recognise the studied properties of a unit,
Data collection	– distinguish between descriptive and qualitative data, cardinal and ordinal as well as numerical and quantitative data,
Management and structuring of data	– collect, manage and structure data,
Data representation (column chart, position chart, pie chart, histogram, scatter plot, line and curve charts, a box plot)	– select the appropriate diagram to represent data,
Arithmetic mean, median, mode	– read, make and interpret statistical diagrams,
Variance, standard deviation, interquartile range	– develop a critical attitude towards the interpretation of results,
Statistical task	– are familiar with and use different methods of summarising data,
	– choose the appropriate method of summarising data with regard to the type of data,
	– calculate, evaluate and interpret the average, the mode and the median as measures of central tendency of data,
	– evaluate simple connections between variables in statistics,

Content	Objectives
	– calculate, evaluate and interpret the variance, the standard deviation and the interquartile range as measures of spread, – apply knowledge on how to use data in a complex procedure of empirical research (choose a topic, specify the research question, collect, manage, structure and analyse data, show and interpret results).

5 REFERENCE MATERIALS

Textbooks and learning tools approved by the Council of Experts of the Republic of Slovenia for General Education are listed in the *Catalogue of Textbooks for Secondary Education* and published on the National Education Institute Slovenia (*Zavod Republike Slovenije za šolstvo*) website www.zrss.si.

6.1 Mathematical symbols

► Logic

$\wedge$, &	conjunction
$\vee$	disjunction
$\Rightarrow$	implication
$\Leftrightarrow$	equivalence
$\neg A, \overline{A}$	negation of statement A
$\forall$	for each
$\exists$	there exists

► Sets

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, ...\}$	the set of elements $x_1, x_2 ...$
$\{x;...\}, \{x \mid ...\}$	the set of all x, so that…
$m(A)$, $\lvert A \rvert$	the number of elements (i.e., power) of the set A
$\mathcal{P}A$, $\mathcal{P}(A)$	the power set of set A
$\varnothing$, { }	the empty set
$\mathcal{U}$	a universal set (a universe)
A^C, A'	the complement of set A
$\mathbb{N} = \{1, 2, 3, ...\}$	the set of positive integers
$\mathbb{N}_0$	$\mathbb{N} \cup \{0\}$
$\mathbb{Z}$	the set of integers
$\mathbb{Z}^+$	the set of positive integers
$\mathbb{Z}^-$	the set of negative integers
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers
$\mathbb{Q}^-$	the set of negative rational numbers
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers
$\mathbb{R}_0^+$	the set of non-negative real numbers
$\mathbb{R}^-$	the set of negative real numbers
$\mathbb{C}$	the set of complex numbers

$\subset$, $\subseteq$	is a subset of
$\not\subset$, $\not\subseteq$	is not a subset of
$\cup$	a union
$\cap$	an intersection
$\times$	a Cartesian product
$\setminus$, $-$	a difference of sets
$[a,b]$	the closed interval $\{x \in \mathbb{R};\ a \leq x \leq b\}$
$[a,b)$	the interval $\{x \in \mathbb{R};\ a \leq x < b\}$
$(a,b]$	the interval $\{x \in \mathbb{R};\ a < x \leq b\}$
(a,b)	the open interval $\{x \in \mathbb{R};\ a < x < b\}$

► Relations and operations

(a,b)	the ordered pair
$=$	is equal to
$\neq$	is not equal to
$\doteq$, $\approx$	is approximately equal to
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to
$+$	plus
$-$	minus
$\cdot$, $\times$	times
:, $\div$	divide
$a \mid b$	a divides b
$D(a,b)$, gcd(a,b)	the greatest common divisor of integers a and b
$v(a,b)$, lcm(a,b)	the least common multiple of integers a and b
$\sum$	the sum symbol
$\lvert a \rvert$	the absolute value of the integer a

► Complex numbers

i	the imaginary unit
Re z	the real part of the complex number z
Im z	the imaginary part of the complex number z
$\lvert z \rvert$	the absolute value of the complex number z
$\overline{z}$, z^*	the complex conjugate of the complex number z

► Geometry. Vectors

$d(A,B)$	the distance between points A and B
$\lvert AB\rvert$	the length of the line segment AB
$\sphericalangle$	an angle
$\triangle$	a triangle shape
$\parallel$	is parallel to
$\perp$	is perpendicular to
$\cong$	is congruent to
$\sim$	is similar to
$\overrightarrow{AB}$, $\vec{a}$	the vector $\overrightarrow{AB}$, the vector $\vec{a}$
$s\vec{a}$	the product of a vector $\vec{a}$ by a number (a scalar) s
$\vec{a}\cdot\vec{b}$	the dot product of vectors $\vec{a}$ and $\vec{b}$
$\vec{i}$, $\vec{j}$, $\vec{k}$	vectors of standard orthogonal basis
$\vec{a}=(a_1, a_2, a_3)$	the vector with coordinates a_1, a_2, a_3
$\lvert\vec{a}\rvert$	the magnitude of vector $\vec{a}$
$\vec{r}_A$	the position vector of a point A
$A(x,y)$	the point A with coordinates x and y
$A(x,y,z)$	the point A with coordinates x, y and z
S, p	the area of a shape
V	the volume of a solid
P	the surface area of a solid

► Functions

$f: A \to B$	f is a transformation (function) which maps from A to B
$x \mapsto f(x)$	f transforms x into $f(x)$
D_f	the domain of function f
Z_f	the range of function f
f^{-1}	the inverse function of function f
$f \circ g$	the composition of functions f and g
$\lim\limits_{x\to a} f(x)$	the limit value of function f as x approaches a
(a_n), $\{a_n\}$	the sequence given by a general term a_n
$\lim\limits_{n\to\infty} a_n$	the limit of a sequence given by a general term a_n
f', $\frac{\mathrm{d}f}{\mathrm{d}x}$	the (first) derivative of a function f
$\int f(x)\,\mathrm{d}x$, $\int f$	the indefinite integral of a function f

$\int_a^b f(x)\,\mathrm{d}x$	the definite integral of a function f with respect from a to b

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P_n	the number of permutations of n elements without repetition
$P_n^{m_1,m_2,\ldots,m_k}$	the number of permutations of n elements with repetition
$n!$	n factorial
V_n^r	the number of variations of n elements with repetition of the order r
${}^{(p)}V_n^r$	the number of variations of n elements with repetition of the order r
$\binom{n}{r}$	the binomial coefficient (n choose r)
C_n^r	the number of combinations between n elements without repetition of the order r
G	a certain event
N	an impossible event
E_1, E_2, E_3, ...	elementary events
A', $\overline{A}$	the complementary event to event A
$A \cup B$, $A + B$	the sum of events A and B
$A \cap B$, $A \cdot B$	the product of events A and B
$A \setminus B$, $A - B$	the difference of events A and B
$A \subset B$	A is a subset of event B
$P(A)$	the probability of event A
$P(A \mid B)$	the probability of event A given B (conditional probability)
$\overline{x}$, μ	the arithmetic mean
σ^2	variance
σ	standard deviation

6.2 Formulas from the Formula Sheet

(***Sum and difference of cubes***) *For any* $a,\ b \in \mathbb{R}$ *the following identities hold true* $\boxed{a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)}$.

(***Euclidean and altitude theorem***) *A right-angled triangle has a hypotenuse* c. *The catheti are* a *and* b. *The altitude on the hypotenuse is* v_c *and the projections of the catheti* a *and* b *on the hypotenuse are* a_1 *and* b_1 *respectively. Then* $\boxed{a^2 = ca_1,\ b^2 = cb_1}$, $\boxed{v_c^2 = a_1 b_1}$.

(***Radii of the inscribed and circumscribed circle of a triangle***) *A triangle has sides* $a,\ b$ *and* c. *The semiperimeter is denoted by* $s = \frac{a+b+c}{2}$. *The area of the triangle is* S. *The radius of the inscribed circle is* r *and the radius of the circumscribed circle is* R. *Then* $\boxed{r = \frac{S}{s}}$ *and* $\boxed{R = \frac{abc}{4S}}$.

(***Heron's formula***) *A triangle has sides* $a,\ b$ *and* c. *The semiperimeter is denoted by* $s = \frac{a+b+c}{2}$. *The area of the triangle is* S. *Then* $\boxed{S = \sqrt{s(s-a)(s-b)(s-c)}}$.

(***Area of a triangle***) *Let* $A(x_1,\ y_1),\ B(x_2,\ y_2)$ *and* $C(x_3,\ y_3)$ *be points on a plane. The area* S *of a triangle with vertices* $A,\ B$ *and* C *is* $\boxed{S = \frac{1}{2}\left|(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)\right|}$.

(***Sphere***) *The surface area* P *and the volume* V *of a sphere with radius* r *are* $\boxed{P = 4\pi r^2,\ V = \frac{4\pi r^3}{3}}$.

(***Trigonometric addition formulas***) *For any* $x,\ y \in \mathbb{R}$ *the following identities hold true* $\boxed{\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y}$, $\boxed{\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y}$.

For any $x,\ y \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + \pi \cdot k;\ k \in \mathbb{Z}\right\}$, *such that* $x + y \neq \frac{\pi}{2} + \pi \cdot k,\ k \in \mathbb{Z}$ *and* $\tan x \tan y \neq -1$, *the following identity holds true* $\boxed{\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}}$.

(***Trigonometric half angle formulas***)

For any $x \in \mathbb{R}$ *the following identities hold true* $\boxed{\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}}$, $\boxed{\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}}$.

For any $x \in \mathbb{R} \setminus \{\pi + \pi \cdot 2k;\ k \in \mathbb{Z}\}$ *the following identity holds true* $\boxed{\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}}$.

(***Ellipse***) *Let* a *and* b $(a > b)$ *be semiaxes of an ellipse on a plane. Linear eccentricity of an ellipse is denoted by* e *and numerical eccentricity of an ellipse is denoted by* ε. *Then* $\boxed{e^2 = a^2 - b^2}$, $\boxed{\varepsilon = \frac{e}{a}}$.

(***Hyperbola***) *Let* a *be a real semiaxis and let* b *be an imaginary semiaxis of a hyperbola on a plane. Linear eccentricity of a hyperbola is denoted by* e *and numerical eccentricity of a hyperbola is denoted by* ε. *Then* $\boxed{e^2 = a^2 + b^2}$, $\boxed{\varepsilon = \frac{e}{a}}$.

(***Parabola***) *A parabola on a plane with an equation* $y^2 = 2px$ *has a focus in point* $\boxed{G\left(\frac{p}{2},\ 0\right)}$. *The equation of the directrix of a parabola is* $\boxed{x = -\frac{p}{2}}$.

(***Arithmetic sequence***) *The sum of the first* n *terms of an arithmetic sequence* (a_n) *is* $\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$.

(***Geometric sequence***) *The sum of the first* n *terms of a geometric sequence* (a_n) *with a common ratio* $q \in \mathbb{R}$ *is* $\boxed{S_n = \frac{a_1(q^n - 1)}{q - 1}}$ *if* $q \neq 1$, *and* $\boxed{S_n = na_1}$ *if* $q = 1$.

(***Limits***) $\boxed{\lim_{n \to \infty}\left(1 + \frac{1}{n}\right)^n = e}$ *and* $\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$.